

Computer Algebra Systems Activity: Patterning

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Topic: Patterning and Algebra

Notes to the Teacher:

a) This activity is designed to use the CAS on the TI-Nspire CAS calculator to enhance understanding and instruction. Other CAS systems may be used in place of the TI-Nspire CAS. All screen shots are from the TI-Nspire CAS.

b) The instructions for the activity assume that the user has some elementary experience with a CAS. Novice users should complete the activity Computer Algebra Systems: An Introduction before attempting this activity.

c) The activity is presented in a **Teacher Version**, with all screen shots and solutions present, as well as a **Student Version**, which can be duplicated and handed out to students.

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TI-Nspire CAS Activity: Patterning

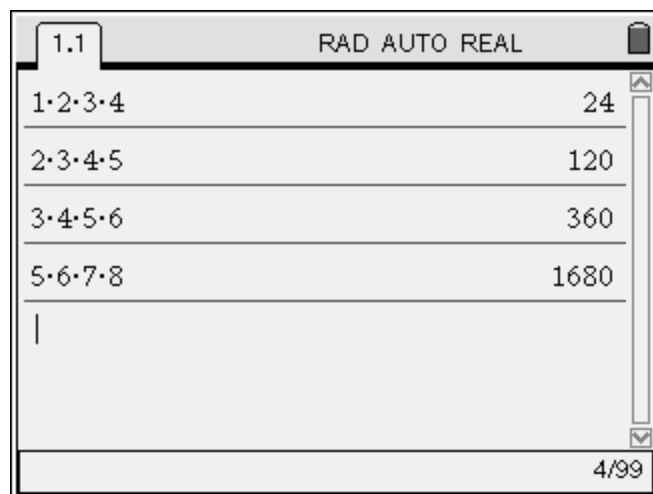
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Teacher Version:

Introduction: Patterns that occur in series of numbers can be determined with pencil and paper or the use of a simple calculator. However, a CAS allows you to verify that your pattern will work for an infinite series. In this exercise, you will use your calculator to help you find a pattern, and then use a CAS to show whether the pattern works regardless of the beginning number chosen.

1. Find the product of the integers from 1 to 4. Then, find the product of the integers 2 to 5. Continue, starting one integer higher each time, until you have five products. Display your results in a table.



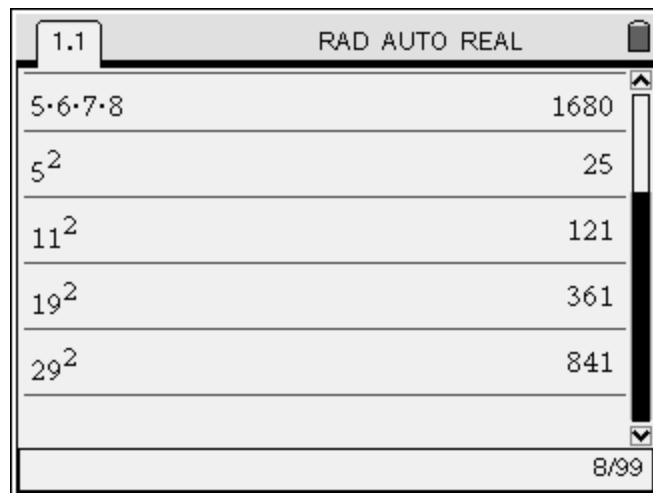
The TI-Nspire CAS screen displays a table with 5 rows and 2 columns. The first column contains the product of consecutive integers, and the second column contains the result. The table is titled '1.1' and 'RAD AUTO REAL'. The results are 24, 120, 360, and 1680. The bottom status bar shows '4/99'.

$1 \cdot 2 \cdot 3 \cdot 4$	24
$2 \cdot 3 \cdot 4 \cdot 5$	120
$3 \cdot 4 \cdot 5 \cdot 6$	360
$5 \cdot 6 \cdot 7 \cdot 8$	1680

2. Look for a pattern in your results. Ensure that each of your results matches the pattern. Then, continue for three more steps, and check the pattern.

(Hint: If you can't find a pattern, decode this message by replacing each letter with the one before it in the alphabet: "dspotjefs trvbsft pg joufhfst")

[Answer: each number is one less than the square of an integer. The base follows the pattern $5 + 2(x - 1)$, where x is the term number.]



The TI-Nspire CAS screen displays a table with 5 rows and 2 columns. The first column contains the square of integers, and the second column contains the result. The table is titled '1.1' and 'RAD AUTO REAL'. The results are 1680, 25, 121, 361, and 841. The bottom status bar shows '8/99'.

$5 \cdot 6 \cdot 7 \cdot 8$	1680
5^2	25
11^2	121
19^2	361
29^2	841

3. For the first term, the product is $1 \times 2 \times 3 \times 4$. Write an expression for the product of the x^{th} term.

[Answer: $x(x + 1)(x + 2)(x + 3)$]

4. Add 1 to this expression.

[Answer: $x(x + 1)(x + 2)(x + 3) + 1$]

5. Define $f(x)$ as the expression in part (4). Verify that $f(x)$ generates the expected numbers for integers from $x = 1$ to $x = 5$.

6. Now you will use the CAS engine to show that the answer will be a perfect square regardless of the integer chosen for x . Select the **3:Expand** function under the **Algebra** menu to expand $f(x)$, and then use the **2:Factor** function to factor the resulting expression.

7. Define $g(x)$ as the factored expression from part (6). Verify that $g(x)$ generates the expected numbers for integers from $x = 1$ to $x = 5$.

1.1	RAD AUTO REAL
29^2	841
Define $f(x)=x \cdot (x+1) \cdot (x+2) \cdot (x+3)+1$	Done
expand($f(x)$)	$x^4+6 \cdot x^3+11 \cdot x^2+6 \cdot x+1$
factor($x^4+6 \cdot x^3+11 \cdot x^2+6 \cdot x+1$)	$(x^2+3 \cdot x+1)^2$
Define $g(x)=(x^2+3 \cdot x+1)^2$	Done
	12/99

1.1	RAD AUTO REAL
Define $g(x)=(x^2+3 \cdot x+1)^2$	Done
$g(1)$	25
$g(2)$	121
$g(3)$	361
$g(4)$	841
$g(5)$	1681
	17/99

8. Extensions:

a) Does the pattern work for four consecutive negative integers?

b) Does the pattern work for a combination of positive and negative integers? Before trying examples on your calculator, do a little thinking, and predict the results. Then, use your calculator to verify the results.

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Student Version:

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1. Find the product of the integers from 1 to 4. Then, find the product of the integers 2 to 5. Continue, starting one integer higher each time, until you have five products. Display your results in a table.

Integers	Product
$1 \times 2 \times 3 \times 4$	

2. Look for a pattern in your results. Ensure that each of your results matches the pattern. Then, continue for three more steps, and check the pattern.

Result	Pattern
$1 \times 2 \times 3 \times 4$	

(Hint: If you can't find a pattern, decode this message by replacing each letter with the one before it in the alphabet: "dpotjefs trvbsft pg joufhfst")

3. For the first term, the product is $1 \times 2 \times 3 \times 4$. Write an expression for the product of the x^{th} term.

4. Add 1 to this expression.

5. Define $f(x)$ as the expression in part (4). Verify that $f(x)$ generates the expected numbers for integers from $x = 1$ to $x = 5$.

6. Now you will use the CAS engine to show that the answer will be a perfect square regardless of the integer chosen for x . Select the **3:Expand** function under the **Algebra** menu to expand $f(x)$, and then use the **2:Factor** function to factor the resulting expression.

7. Define $g(x)$ as the factored expression from part (6). Verify that $g(x)$ generates the expected numbers for integers from $x = 1$ to $x = 5$.

8. Extensions:

a) Does the pattern work for four consecutive negative integers?

b) Does the pattern work for a combination of positive and negative integers? Before trying examples on your calculator, do a little thinking, and predict the results. Then, use your calculator to verify the results.